Metastable dynamical transient states

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We study metastable dynamical transient states (dynamical metastability, metastable dynamics) in various coupled dynamical systems, e.g., neural network models. In systems with metastable dynamical transients, the duration of transient states until the systems reach their asymptotically stable states eventually increases exponentially with the system size, e.g., the number of elements. Consequently, the systems never reach their asymptotically stable states in a practical time when the system size is large so that their metastable dynamical transient states are of practical importance for the functions of the systems.

We have shown that metastable dynamical transient states emerge in a ring of unidirectionally coupled sigmoidal neurons in Eq. (1)

$$dx_n/dt = -x_n + f(gx_{n-1}), \qquad f(x) = \tanh(x) \quad (1 \le n \le N, \quad x_{n\pm N} = x_n, \quad g > 1) \tag{1}$$

This system is bistable and has a pair of asymptotically stable spatially uniform steady states, in which the states of the all neurons take the same positive or negative values ($x_n = f(x_n)$, $1 \le n \le N$). In the transient states, however, the neurons are divided into two bumps in which their states take positive (black) and negative (white) values. Then, the bumps rotate in the coupled direction so that each neuron oscillates, as shown in Fig. 1. When random values are given to the initial states of neurons, the system reaches to one of spatially uniform steady states quickly like (a) or stays in the transient state and the oscillations of neurons sustain for a long time like (b).

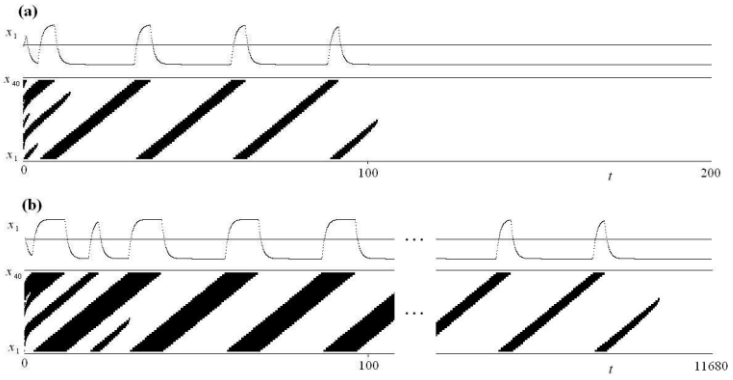

Fig. 1. Spatiotemporal patterns of metastable dynamical transient rotating waves in a ring neural network.

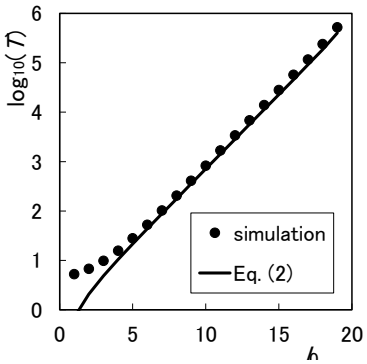

Fig. 2. Duration T of transient rotating waves.

We have proven that changes in the spatial length l of the bump (the number l of neurons in the bump, the vertical width of black stripes in Fig. 1) are expressed by Eq. (2).

$$dx_n/dt = \beta\{\exp[-\alpha(L-l)] - \exp(-\alpha l)\} \quad (\alpha > 0, \quad \beta > 0, \quad 0 < l < L) \tag{2}$$

The analytical solution to Eq. (2) shows that the duration T of transient rotating waves increases exponentially with the initial length $l(0) = l_0$ of the bump and agrees with the results of simulation of Eq. (1). We have also observed several thousand times of oscillations in an analog circuit of Eq. (1) consisting of forty neurons made with operational amplifiers. Further, we have shown that such metastable dynamical transient waves emerge in various unidirectionally coupled systems such as Duffing elements, Bonhoeffer-van der Pol(BVP) neurons, parametric oscillators and the Lorenz models.